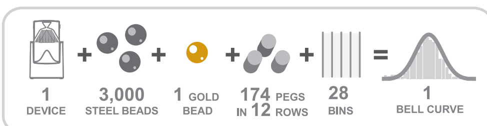
GALTON BOARD

DESKTOP EDITION

Math in Motion

The Galton Board sets *math in motion*, demonstrating centuries-old mathematical concepts in an innovative desktop device. It incorporates Sir Francis Galton's (1822-1911) illustration of the binomial distribution, which for a large number of beads approximates the normal distribution. **Pascal's triangle** (Blaise Pascal, 1623-1662) is overlaid on the pegs. This triangle of numbers generally follows the rule of adding the two numbers above to get the number below. The number corresponding to each peg represents the number of paths a bead can travel from the top peg to that particular peg. It also shows the **Fibonacci numbers** (Leonardo Fibonacci, 1175-1250), which are the sums of specific diagonals in Pascal's triangle.



When the Galton Board is rotated on its axis, 3,000 steel beads cascade through rows of symmetrically placed pegs. If the device is level, each bead bounces off the pegs with equal probability of moving to the left or right, which represents a **binomial distribution** (n, p) described by a number of trials (n), equal to the number of rows on the board and a probability of success (p), which is equal to 0.5. As the beads settle into the bins at the bottom of the board, they accumulate to approximate a bell-shaped curve. The bell curve, also known as the **Gaussian distribution** (Carl Friedrich Gauss, 1777-1855), is important in statistics and probability theory. It is used in the natural and social sciences to represent random variables, like the beads in the Galton Board.

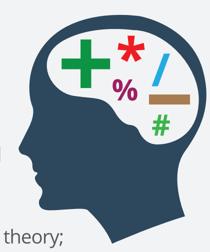
To illustrate the concept of a single random outcome, we have included one gold bead. Be aware that it may be very hard to find it among the 3,000 steel beads. Can you guess which the numbered bins the gold bead is likely to land in 68% of the time?

STEM.ORG AUTHENTICATED™ EDUCATIONAL PRODUCT

STEM stands for **S**cience, **T**echnology, **E**ngineering and **M**ath. STEM education integrates concepts that are usually taught as separate subjects in different classes and emphasizes the application of knowledge to real-life situations. The Galton Board has been authenticated by STEM.org for STEM educational activities.

Download the STEM Curriculum at GaltonBoard.com/STEM

SET YOUR MATH BRAIN IN MOTION!



Printed on the Galton Board is Pascal's triangle, the formula for the normal distribution, binomial expressions, Fibonacci numbers, Powers of 2, Powers of 11, the Hockey Stick Pattern, the Star of David theorem, the bell curve with standard deviation lines, and numbered bins with the expected percentages of beads and the expected numbers of beads.

Embedded in this Galton Board are many statistical and mathematical concepts: including probability theory; independent identically distributed (IID) random variables; the Gaussian or normal distribution; bell-shaped curves; the central limit theorem (the de Moivre–Laplace theorem); Bernoulli distribution (Jacob Bernoulli, 1655–1705), which is a special case of the binomial distribution; regression to the mean; probabilities such as coin flipping and stock market returns; the law of frequency of errors; and what Sir Francis Galton referred to as the "law of unreason."

"The Galton Board is a chilling reminder that out of wonderful, wild randomness, order and stability can emerge." - **Michael Stevens**

Visit galtonboard.com/video to watch Michael's video

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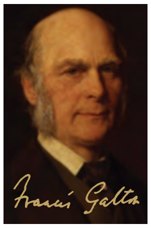
Sir Francis Galton Describes His Board



Sir Francis Galton

In his book *Natural Inheritance* (1889), Sir Francis Galton colorfully described the apparatus he created to reveal the order in apparent chaos. Below is a modified excerpt from that book. The text has been updated to correspond to the terminology used to describe the current Galton Board.

The Charms of Statistics

"It is difficult to understand why statisticians commonly limit their inquires to Averages, and do not revel in more comprehensive views. Their souls seem as dull to the charm of variety as that of the native of one of our flat English counties, whose retrospect of Switzerland was that, if its mountains could be thrown into its lakes, two nuisances would be got rid of at once. An Average is but a solitary fact, whereas if a single other fact be added to it, an entire Normal Scheme, which nearly corresponds to the observed one, starts potentially into existence.

Some people hate the very name of statistics, but I find them full of beauty and interest. Whenever they are not brutalized, but delicately handled by the higher methods, and are warily interpreted, their power of dealing with complicated phenomena is extraordinary. They are the only tools by which an opening can be cut through the formidable thicket of difficulties that bars the path of those who pursue the Science of man."

Mechanical Illustrations of the Cause of the Curve of Frequency

"The Curve of Frequency, and that of Distribution, are convertible: therefore, if the genesis of either of them can be made clear, that of the other becomes also intelligible. I shall now illustrate the origin of the Curve of Frequency, by means of an apparatus that mimics in a very pretty way the conditions on which Deviation depends."

Our design of the Galton Board is constructed of a plastic frame. A reservoir is designed into the top of the board. Below the outlet of the funnel stands a succession of rows of pegs stuck squarely into the back of the board, and below these again are a series of bins, or vertical compartments. A charge of 3,000 steel beads are enclosed in the board. When the board is flipped "topsy-turvy", all the beads run to the upper end into the reservoir; then, when it is turned back into its working position, the desired action commences. The borders of the reservoir have the effect of directing all the beads that had collected at the upper end of the frame to run into the mouth of the funnel.

"The beads pass through the funnel and scamper deviously down through the pegs in a curious and interesting way; each of them darting a step to the right or left, as the case may be, every time it strikes a peg. The pegs are disposed in a quincunx fashion, so that every descending bead strikes against a peg in each successive row. The cascade issuing from the funnel broadens as it descends, and, at length every bead finds itself caught in a bin immediately after freeing itself from the last row of pegs. The outline of the distribution of beads that accumulate in the bins approximates to the Curve of Frequency, and is closely of the same shape however often the experiment is repeated."

"The principle on which the action of the apparatus depends is, that a number of small and independent accidents befall each bead in its career. In rare cases, a long run of luck continues to favor the course of a particular bead towards either outside bin, but in the large majority of instances the number of accidents that cause Deviation to the right, balance in a greater or less degree those that cause Deviation to the left. Therefore most of the beads find their way into the bins that are situated near to a perpendicular line drawn from the outlet of the funnel, and the Frequency with which beads stray to different distances to the right or left of that line diminishes in a much faster ratio than those distances increase."

Order in Apparent Chaos

"I know of scarcely anything so apt to impress the imagination as the wonderful form of cosmic order expressed by the "Law of Frequency of Error." The law would have been personified by the Greeks and deified, if they had known of it. It reigns with serenity and in complete self-effacement amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason. Whenever a large sample of chaotic elements are taken in hand and marshaled in the order of their magnitude, an unsuspected and most beautiful form of regularity proves to have been latent all along. The tops of the marshaled row form a flowing curve of invariable proportions; and each element, as it is sorted into place, finds, as it were, a pre-ordained niche, accurately adapted to fit it. If the measurement at any two specified Grades in the row are known, those that will be found at every other Grade, except towards the extreme ends, can be predicted in the way already explained, and with much precision."

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Normal Distribution Formula

In probability theory, a normal distribution is a type of continuous probability distribution for a real-valued random variable. Shown here is the general form of its probability density function ($f(x)$). Normal distributions are important in statistics and are often used in the natural and social sciences to represent real-valued random variables whose distributions are not known. Included in the formula is the constant Pi ($\pi \approx 3.142$) which is the ratio of a circle's circumference to its diameter. Also included is Euler's number ($e \approx 2.718$) which is the base of the natural logarithm. The Independent Identically Distributed (IID) Central Limit Theorem states that the random variable x will be normally distributed as the sample size becomes large and sigma is finite.

Powers of 11

If you collapse each row into a single number by taking each element as a digit (and carrying over to the left if the element has more than one digit) you get the powers of eleven: **1, 11, 121,1331, 14641, 161051**...

Powers of 2

The sum of numbers in a row is equal to 2^n where n equals the row number.

Star of David Theorem

The Star of David theorem says the two sets of three numbers surrounding a number have equal products. In the example shown, the number **4** is surrounded by, in sequence, the numbers **1, 3, 6,10, 5, 1**, and taking alternating numbers, we have **1×6×5** = **3×10×1**.

Hockey Stick Pattern

The sum of the numbers in a diagonal, starting from the edge with 1, is equal to the number in the next diagonal below. Outlining these numbers reveals a hockey stick pattern, as seen here in **1** + **9** + **45** = **55**.

Bin Numbers, Expected Percentages and Beads

Each bin is numbered so it be can easily identified. Under the bin numbers are the expected percentage of outcomes (**%**) and the expected number of beads (**#**) for that bin, based on 3,000 beads.

Steel Beads

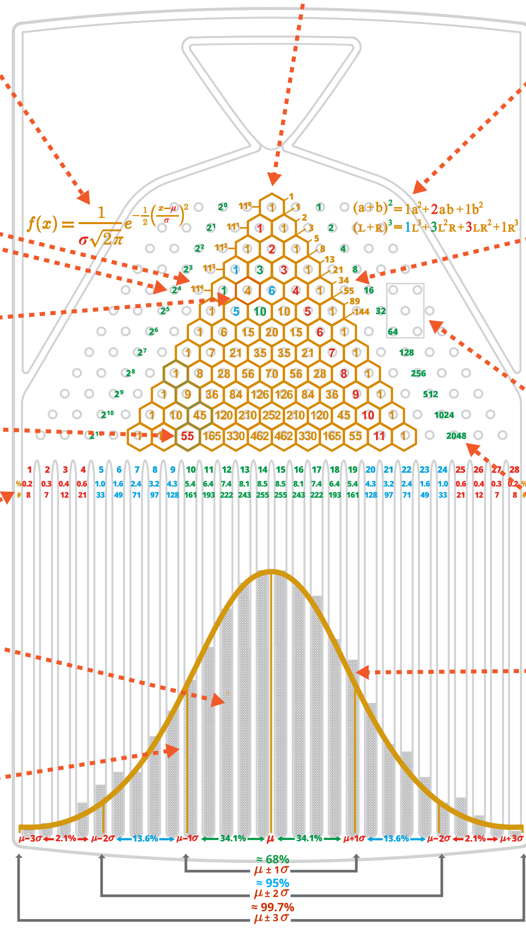
Each 1 mm steel bead, including one gold bead, represents an independent and identically distributed (IID) random variable that falls from the reservoir through a fixed pattern of pegs. When all 3,000 steel beads are collected in the bins they form a similar distribution every time. The discrete binomial distribution of beads closely approximates the continuous normal distribution.

Standard Deviation Line

The standard deviation (σ) is a measure of how closely all of the data points are gathered around the mean (μ). The shape of a normal distribution is determined by the mean and the standard deviation. About 68% of the data (beads) in a bell curve fall within one standard deviation of the mean. About 95% fall within two standard deviations and about 99.7% fall within three standard deviations.

Pascal's Triangle

Pascal's Triangle is a triangle of numbers that follow the rule of adding the two numbers above to get the number below. This pattern can continue endlessly. Blaise Pascal (1623-1662) used the triangle to study probability theory, as described in his mathematical treatise *Traité du triangle arithmétique* (1665). The triangle's patterns translate to mathematical properties of the binomial coefficients.



Binomial Theorem

The Binomial Theorem describes the algebraic expansion of powers of a binomial. Pascal's triangle defines the coefficients which appear in binomial expansions. That means the n^{th} row of Pascal's triangle comprises the coefficients of the expanded expression of the polynomial $(a+b)^n$. For the Galton Board, the binomials are left and right $(L+R)^n$.

The expansion of $(a+b)^n$ is: $(a+b)^n = x_0a^n + x_1a^{n-1}b + \ldots + x_{n-1}ab^{n-1} + x_nb^n$ where the coefficients of the form x_k are precisely the numbers that appear in the k^{th} entry of the n^{th} row of Pascal's triangle (k and n counting starts at 0). This can be expressed as: $x_k = \binom{n}{k}$, pronounced "n choose k". The first peg on Galton Board is $\binom{0}{0}$, followed below by $\binom{1}{0}$ and $\binom{1}{1}$. Examples of binomial expressions are shown for $(a+b)^n$ for n = 2 and $(L+R)^n$ for n = 3. The numbers in each hexagon are the number of paths that a bead can take to arrive at that location.

Fibonacci Numbers and the Golden Ratio

The sum of the numbers on the diagonal shown on Pascal's Triangle match the Fibonacci numbers. The sequence progresses in this order: **1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89** and so on. Each number in the sequence is the sum of the previous two numbers. For example: 2+3=5; 3+5=8; 5+8=13; 8+13=21 and so on. Leonardo Fibonacci popularized these numbers in his book *Liber Abaci* (1202). As you progress through the Fibonacci numbers, the ratios of consecutive Fibonacci numbers approach the Golden Ratio of 1.61803398...., but never equals it. For example: 55/34=1.618; 89/55=1.618 and 144/89=1.618. This Galton Board has a length to width ratio that approximates the Golden Ratio.

Quincunx Pattern

The pegs on the board are in a Quincunx pattern, which is an arrangement of five objects with four at the corners of a square or rectangle and the fifth at its center.

Row Totals

The sum of the numbers in each row is shown here and each total doubles on subsequent rows.

Bell Curve

The normal distribution, often referred to as the "bell curve", is the most widely known and used of all probability distributions. Because the normal distribution approximates many natural phenomena so well, it has developed into a common reference for numerous probability problems. The normal distribution can often be used to model data sets such as the heights of adults, the weights of babies, classroom test scores, returns of the stock market and the beads in the Galton Board.

X-axis

This scale provides the mean and 3 standard deviations from the mean.

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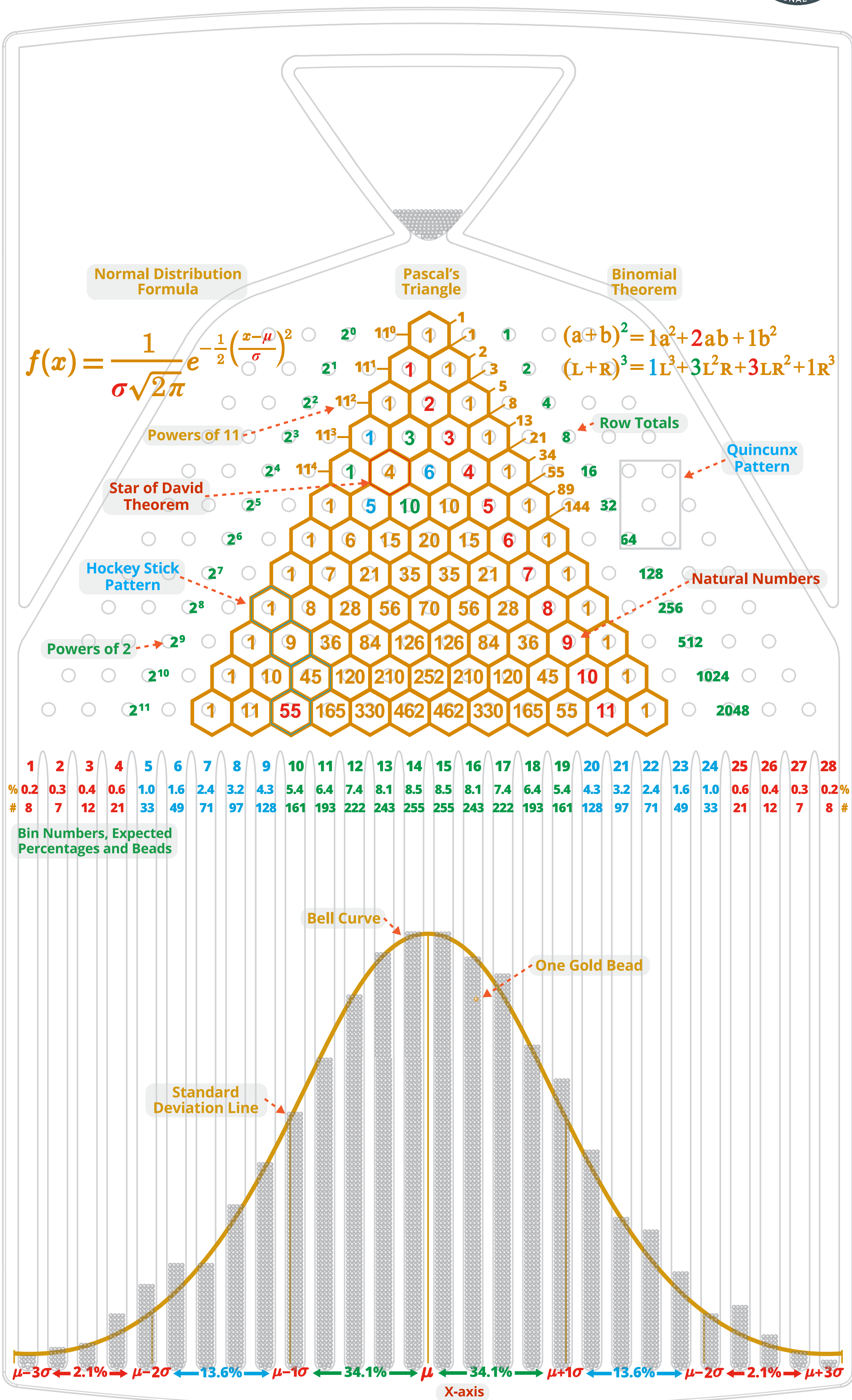

Our Galton Board is a desktop design reminescent of Charles and Ray Eames' groundbreaking 11-foot-tall "Galton's Probability Board," featured at the 1961 ***Mathematica: A World of Numbers . . . and Beyond*** exhibit, which is currently on display at the Museum of Science in Boston, Massachusetts; the New York Hall of Science, in Queens, New York; and most recently, The Henry Ford Museum in Dearborn, Michigan. ***Mathematica*** was sponsored by IBM and is celebrating its 60th Anniversary. On the side of the machine is this quote: "*Probabilities is nothing more than good sense confirmed by calculations" - LaPlace 1796*. Pictured below is an approximately 4-foot-tall information sign from the ***Mathematica*** Exhibit (some of the text has been enlarged so it is legible at this size). An even larger 14 1⁄2-foot-tall Eames Probability Board was showcased at IBM's Pavilion for the 1964 World's Fair in New York. Seeing this amazing machine in an Eames film about the 1964 World's Fair was the inspiration for making our Galton Board.